

## RELATIVE VISCOSITY OF FLUIDIZED BED

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Relative viscosity of a fluidized bed at fluidization by gases was determined from relations obtained on basis of the assumption of analogy between the liquid flow in open channels and the flow of a fluidized bed in airlifts. Relative viscosities of fluidized beds in dependence on the relative velocity of the fluidizing fluid (ratio of gas velocity to the velocity at minimum fluidization) and on dimensionless velocity (ratio of difference of gas velocity and velocity at minimum fluidization to the difference of velocity at minimum elutriation and velocity at minimum fluidization) were obtained on basis of our own experiments as well as of data given in literature. Experimental data were compared with the relative viscosity of water in dependence on temperature. A relatively good agreement enabled derivation of equations for the relative viscosity of a fluidized bed in dependence on the relative or dimensionless gas velocity.

Viscosity of a fluidized bed and the analogy between the flow of the fluidized bed and the Newtonian liquid is studied. Viscosity of disperse systems, in our case of the fluidized bed  $\eta_B$ , is sometimes defined formally in the same way as with Newtonian liquids for the undimensional flow by the relation

$$\tau = \eta_B \, du_B/dy \quad (1)$$

where  $du_B/dy$  is the projection of the velocity gradient in the direction perpendicular to the direction of the mean flow. Differential length  $dy$  must be understood here sufficiently long in comparison to the mean distance of the nearest solid particles and simultaneously sufficiently short in comparison to the characteristic linear dimension of the system. The quantity  $u_B$  is the local velocity of the fluidized bed and at fluidization by gases it is practically identical with the local velocity of granular material  $u_s$ .

With regard to the macroscopic character of solid particles in the fluidized bed the quantity  $\eta_B$  in Eq. (1) is, however, the apparent and not according to definition molecular viscosity. But nevertheless, introduction of this definition could be advantageous since it enables description of the fluidized bed.

A fully satisfactory method for determination of viscosity of the fluidized bed is not yet available. The published methods for measurement of this quantity are mostly

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only applications of methods used in measurements of viscosities of homogeneous fluids. They are mostly the method of rotation of the body submerged into the fluidized bed<sup>1-5</sup> and the method of fall of a solid sphere in the fluidized bed<sup>6-8</sup>.

Application of these methods for the fluidized bed is rather disputable. In measurements the natural fluidized state should be disturbed as little as possible. At rotation of blades of the viscometer or at the fall of a sphere such disturbing effects obviously take place. The results can be also affected by turbulence in the bed which is the result of a rapid motion of a foreign body. To obtain the correct results (with the usual relations), it is necessary that the measured medium stick to the surface of the indicating body. This condition can be approached if the ratio of diameters of the indicating and solid particles in the bed is sufficiently large<sup>7</sup>. This ratio is in fluidized beds studied in laboratories usually smaller. Porosity of the gas fluidized bed is changing in the cross-section as well as along the height<sup>9</sup>. It is possible to expect differences in viscosities according to the size and location in the fluidized bed in which the measurement is performed.

For these reasons the procedure used by Siemens and Hellmer<sup>10,11</sup> and authors<sup>12</sup> seems to be the most accurate method of determination of viscosity of flowing fluidized beds. The authors<sup>10-12</sup> have determined the viscosity of the fluidized bed on basis of its flow in the airslide. A considerable advantage of this method is that the fluidized bed is not influenced by the presence of measuring bodies. The same advantage has the method used in our measurements<sup>13-15</sup>.

Our procedure of measurement of the fluidized bed viscosity is based similarly as the method used by authors<sup>10,11</sup> on the assumption of an analogy between the liquid flow in open channels and the flow of the fluidized bed in airslides. Both these methods differ in the approach to the solution. Siemes and Hellmer<sup>10,11</sup> determine the absolute viscosity from the equation for laminar mass flow rate of liquid in open channels. The flow is considered to be laminar on basis of visual observations of the velocity profile of particles as well as from the value of the Reynolds number. But at the same time the viscosity needed for calculation of this number is determined from the relation which is also only considered to be suitable for description of flow of the fluidized bed. On the contrary, the authors<sup>12</sup> have determined the form of dependence of shear stress on the velocity gradient and they have made conclusions from this relation on behaviour of the fluidized bed. They have found that the flow of the fluidized bed is affected not only by properties of particles and the velocity of fluidizing fluid but also by dimensions of the experimental apparatus. In wide and shallow beds at large velocities of the fluidizing fluid the fluidized bed appeared to be a dilatant liquid. Narrow and high beds have had on the contrary, pseudoplastic properties. Between these two extremes the fluidized bed behaved as the Newtonian liquid. Our preliminary results<sup>13,14</sup> have pointed that in the range of our experimental conditions it is possible to consider the flow of

the fluidized bed approximately analogic to the flow of the Newtonian liquid. But in our study we have, unlike those authors<sup>12</sup>, concentrated ourselves to very shallow beds. Thus it is very probable that in the case of the considered study<sup>12</sup>, the given difference is due to bubbles, as the effect of bubbles on flow can be expected first of all with high beds.

### THEORETICAL

We start with the assumption of an analogy between the flow of liquid in open channels and the flow of the fluidized bed in the airslide. Equations are used for description of flow of the fluidized bed which are valid for steady uniform liquid flow in open channels *i.e.* for flow which is at constant cross-sectional area characterised by a constant height  $h$  of liquid along the channel<sup>16,17</sup>:

$$\dot{m}_l = bh\rho_l(8g/f_l)^{1/2} (r_h \sin \beta)^{1/2}. \quad (2)$$

Under the above discussed assumption of an existing analogy between the both flows, we can substitute for physical quantities in equations of liquid flow the corresponding quantities for the fluidized bed (fluidization by gas).

We obtain a general equation for the mass flow rate of granular material  $\dot{m}_s$  through the airslide<sup>13</sup>

$$\dot{m}_s = bh\rho_B(8g/f_B)^{1/2} (r_h \sin \beta)^{1/2}, \quad (3)$$

$$r_h = (bh)/(b + 2h). \quad (4)$$

Eq. (3) can be considered the definition equation for the friction factor of the fluidized bed  $f_B$ . At liquid flow in channels the friction factor in the turbulent region is constant for the given roughness. In the laminar flow region it is a function of the Reynolds number<sup>17,18</sup>.

Similarly as with fluids, we express the friction factor for the fluidized bed by the equation

$$f_B = A \text{Re}_B^a, \quad (5)$$

$$\text{Re}_B = (4\rho_B r_h \bar{u}_s)/\eta_B \quad (6)$$

where  $a$  and  $A$  are constants in a certain flow region.

To avoid the problematic determination of the absolute viscosity of the fluidized bed, we introduce in Eq. (6) the quantity  $R_B$  which is the product of the Reynolds number  $\text{Re}_B$  and of viscosity  $\eta_B$  in the form:

$$R_B = \text{Re}_B \eta_B = 4r_h \rho_B \bar{u}_s. \quad (7)$$

The quantity  $R_B$  includes only the well measurable variables.

Then the relation (5) can be written in the form

$$f_B = BR_B^a, \quad (8)$$

$$B = A/\eta_B^a. \quad (9)$$

Viscosity of the fluidized bed is dependent on the rate of fluidizing fluid (source of particle momentum) which results both from the made assumptions and from practice. Thus we consider the dependence of the fluidized bed viscosity  $\eta_B$  on the relative velocity of the fluidizing fluid  $W$ , defined as the ratio of superficial velocity of the fluidizing fluid  $w$  and minimum fluidization velocity of fluid  $w_p$ , which can be written in the form:

$$\eta_B = \eta_B(W). \quad (10)$$

Relative velocity  $W$  is more suitable than the superficial velocity  $w$  as this reduces the number of independent variables.

With regard to relation (10) also for the quantity  $B$  holds the dependence

$$B = B(W).$$

We choose some reference relative velocity  $W = W_0$  at which the viscosity of the fluidized bed has the value  $\eta_{B0}$  and define the relative viscosity of the fluidized bed  $E_B$  by the relation

$$E_B = \eta_B/\eta_{B0}. \quad (11)$$

Relative viscosities  $E_B$  are determined according to Eqs (9) and (11) from the relation

$$E_B = (B_0/B)^{1/a} \quad (12)$$

where  $B_0$  is the quantity  $B$  at the reference velocity  $W_0$ . The results enable to find the concrete form of the dependence

$$E_B = E_B(W). \quad (13)$$

Let us consider the analogy of dependence of the fluidized bed viscosity on the fluidizing fluid velocity and of the Newtonian liquids viscosity  $\eta_1$  on liquid temperature  $t$  (the source of momentum of molecules) which is

$$\eta_1 = \eta_1(t). \quad (14)$$

Similarly as with the fluidized bed let us define the relative viscosity of the Newtonian liquid  $E_1$

$$E_1 = \eta_1/\eta_{10}, \quad (15)$$

where  $\eta_{10}$  is the viscosity of the considered liquid at the chosen reference liquid temperature  $t = t_0$ . For the relative viscosity of the liquid  $E_1$  also holds

$$E_1 = E_1(t). \quad (16)$$

From the condition of an analogy<sup>19</sup> of relations (13) and (16) the linear transformation is obtained

$$(t - t_0)/(W - W_0) = K. \quad (17)$$

The relative velocity of fluid which is used as the argument in Eq. (10) is relatively easily determined. It varies for the fluidized bed in the range  $W \in <1; W_e$  where  $W_e$  is the relative fluid velocity at incipient elutriation. This velocity is different for various materials and it is in general, a function of the Archimedes number. According to the graph in literature<sup>20</sup> considerable changes in the elutriation velocity of materials take place mostly in the so-called transition region.

In this range of Archimedes numbers it is probably more suitable to use as the independent variable the dimensionless velocity  $U$  defined by the relation

$$U = (w - w_p)/(w_e - w_p). \quad (18)$$

The quantity  $U$  varies for all materials in the range  $U \in <0; 1$ ). This is the advantage in comparison to the use of relative velocity  $W$ . The disadvantage of quantity  $U$  is its less accurate and more cumbersome determination.

The transformation between the temperature  $T$  and the quantity  $U$  is similar to the relation (17)

$$(t - t_0)/(U - U_0) = K', \quad (19)$$

where  $U_0$  is the value of  $U$  corresponding to the velocity of the fluidizing fluid at which  $\eta_B = \eta_{B0}$ .

Transformation relations (17) and (19) make possible a unique subjunction of temperature to the relative velocity  $W$  of the fluidizing fluid or to the velocity  $U$ . At the validity of the mentioned analogy this enables to express, from the known dependence (14), the concrete form of relations (13) or of the relation

$$E_B = E_B(U). \quad (20)$$

## EXPERIMENTAL

For the experiments, the same apparatus was used as that described in literature<sup>21,22</sup>. The grid of the airslide had the dimensions 808 × 43 mm. In the experiments granulated corundum was used with the equivalent particle diameter 0.5 mm.

The measurements were performed at constant angle of inclination of the grid 3° and at relative air velocities  $W = 1.3; 1.4; 1.5; 1.7; 1.8; 1.9; 2.0; 2.3; 2.5$  as well as at constant air velocities  $W = 1.7$  with the angle of inclination of the grid 1, 3, and 5°. The maximum height of the fluidized bed was 42 mm.

Experimental results are given in our last publication<sup>22</sup>. The total number of experiments performed is 286.

## RESULTS AND DISCUSSION

The friction factor  $f_B$  was calculated on basis of the experimental data by Eq. (3), the quantity  $R_B$  by Eq. (7). The calculated values of these quantities were given in our last paper<sup>22</sup>.

The dependence of the friction factor  $f_B$  on quantity  $R_B$  (at  $W = \text{const}$ ) can be expressed in logarithmic coordinates by a straight line (see papers<sup>13,15,22</sup> and Fig. 1) which suits the Eq. (8). The constants  $a$  and  $B$  in this equation were obtained from value of the friction factor  $f_B$  and quantity  $R_B$  by the method of least squares<sup>23</sup>. The values were in individual dependences expressed according to the velocity of air and angle of inclination of the grid.

The determined values of constants  $a$  and  $B$  in Eq. (8) are summarized in Table I. In this Table are also given limits of confidence of linearized relations  $t_{\alpha S_{\log f}}$ , where

TABLE I  
Constants in Eq. (8) for Corundum

$-a$	$B$	$W$	$\beta$	$t_{\alpha S_{\log f}}$
1.77	182.5	1.3	3	0.073
1.55	121.4	1.4	3	0.094
1.49	85.4	1.5	3	0.077
1.41	83.5	1.7	3	0.12
1.47	62.0	1.8	3	0.12
1.51	54.6	1.9	3	0.066
1.48	48.5	2.0	3	0.080
1.37	33.6	2.3	3	0.10
1.39	35.6	2.5	3	0.10
1.45	51.9	1.7	1	0.099
1.45	58.4	1.7	5	0.14

$s_{\log f}$  is the standard deviation of the quantity  $\log f_B$  and  $t_x$  is the parameter of the Student's distribution which is applied with a 5% statistical significance. In agreement with the considerations made in the theoretical part of this study, the quantity  $B$  is dependent on velocity of air. The exponent  $a$  is practically independent on the velocity of air. Significantly deviates only the value for  $W = 1.3$ . At this small velocity of air in vicinity of incipient fluidization obviously local fluidization disturbances take place. As the consequence, the results concerning the flow can be distorted.

From the known values of constants  $a$  and  $B$  the relative viscosity of the fluidized bed  $E_B$  could have been calculated according to the relation (12). The relative velocity of air  $W_0 = 2.5$  was chosen. As the reference, the value of  $B_0$  was for this velocity obtained from the smoothed dependence  $B = B(W)$ . So the value of the constant  $B_0 = 32$  was determined. The exponent  $a$  needed for calculation of the quantity  $E_B$  is determined as the arithmetic mean of all its values except for  $W = 1.3$ . So determined arithmetic mean of exponents equals  $a = -1.46$ .

The relative viscosities of the fluidized bed  $E_B$  calculated according to Eq. (12) on basis of our experiments and from the values taken from literature<sup>1,3,6,10,11</sup> are summarized in Table II.

The plots of relative viscosities of the fluidized bed  $E_B$  in dependence on relative velocity of air  $W$  is compared with the plots of relative viscosity of water  $E_1$  in dependence on temperature  $t$  in Fig. 2. The full points are relative viscosities of corundum, other points correspond to data given in literature and the solid line represents the dependence of relative viscosity of water.

The reference temperature  $t_0 = 60^\circ\text{C}$  was chosen for water. The relative viscosities of water are given in Table III. Dynamic viscosities of water necessary for their determination are taken from literature<sup>24</sup>. In Fig. 2 there are the temperature and velocity scales plotted so that the zero of the temperature scale is identical with the value of  $W$  at the minimum fluidization *i.e.* at  $W = 1$ . There are in correspondence

FIG. 1  
Friction Factor  $f_B$  in Dependence on Quantity  $R_B$   
— according to Eq. (8) and Table I,  
---- limits of confidence,  $\circ$   $W = 2.0$ ;  
 $\bullet$   $W = 1.4$ .

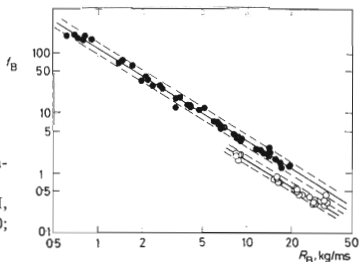


TABLE II

Relative Viscosity of Fluidized Bed  $E_B$  from Our Experimental Measurements and According to Literature

Author Material Method of measurement	$W$	$E_B(W)$	$U$	$E_B(U)$
Our experimental data	1.3	3.30	0.015	3.82
Corundum; $d = 0.5$ mm	1.4	2.49	0.020	2.90
Flow of fluidized bed in airslide	1.5	1.96	0.025	2.31
	1.7 ( $\beta = 1^\circ$ )	1.39	0.033	2.30
	1.7 ( $\beta = 3^\circ$ )	1.93	0.033	2.30
	1.7 ( $\beta = 5^\circ$ )	1.51	0.033	2.30
	1.8	1.57	0.037	1.96
	1.9	1.44	0.042	1.81
	2.0	1.33	0.047	1.70
	2.3	1.03	0.061	1.36
	2.5	1.07	0.070	1.43
Kramers <sup>3</sup>	1.6	2.00	—	—
Sand; $d = 0.127$ mm	1.7	1.63	—	—
Rotating dumbbells	2.0	1.17	—	—
	2.3	1.07	—	—
	2.5	1.00	—	—
	2.7	1.07	—	—
Matheson, Herbst	2.5	1.00	0.042	2.56
Holt <sup>1</sup>	3.6	0.54	0.073	1.38
Catalyst; $d = 0.298$ mm	5.5	0.31	0.126	0.80
Rotary viscometer	7.4	0.27	0.179	0.70
	9.8	0.21	0.249	0.54
Peters, Schmidt <sup>6</sup>	3.8	0.55	0.060	1.33
Sand; $d = 0.125$ mm	5.3	0.40	0.092	1.06
Fall of spheres	7.1	0.33	0.130	0.81
	9.2	0.28	0.176	0.71
Siemes, Hellmer <sup>10</sup>	2.5	1.00	0.041	3.72
Sand; $d = 0.200$ mm	2.8	0.72	0.050	2.67
Flow of fluidized bed in airslide	3.5	0.48	0.070	1.78
	4.2	0.33	0.089	1.22
	5.0	0.24	0.108	0.89
	5.6	0.23	0.128	0.89

also the reference temperature  $t_O$  and the reference relative velocity of air  $W_O$  i.e. the values  $t_O = 60^\circ\text{C}$  and  $W_O = 2.5$ .



TABLE III  
Relative Viscosity of Water  $E_1$

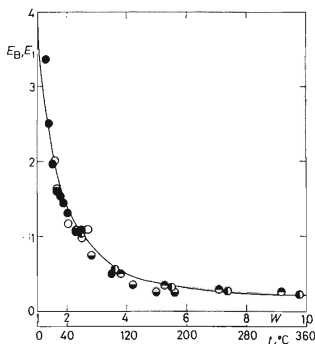
$t$ °C	$E_1$	$t$ °C	$E_1$	$t$ °C	$E_1$	$t$ °C	$E_1$
0	3.80	50	1.17	100	0.60	200	0.29
10	2.76	60	1.00	120	0.49	250	0.23
20	2.12	70	0.86	140	0.42	300	0.20
30	1.70	80	0.76	160	0.37	350	0.16
40	1.39	90	0.67	180	0.33	—	—

It is also obvious from Fig. 2 that with the exception of the dimensionless viscosity  $E_B$  at  $W = 1.3$  the experimental viscosities of the fluidized bed of corundum are in good correspondence with the relative viscosities of water. For comparison there are in Fig. 2 also plotted the viscosities  $E_B$  calculated from measurements of other authors<sup>1,3,6,10,11</sup> who have used various method for determination of viscosity. These data are given in Table II together with information on the used type of material, size of particles and method of viscosity measurement. In all cases the relative velocity  $W_0 = 2.5$  as the reference was chosen. Only for the viscosity determined by Peters<sup>6</sup> who has measured at higher relative velocities of air the viscosity  $\eta_{B0}$  was chosen so that  $E_B = E_1$  at the lowest given value  $W = 3.8$ .

Also the relative viscosities obtained from literature data which are plotted in Fig. 2 are in a good agreement with the dependence of the relative viscosity of water

FIG. 2  
Relative Viscosity of Water  $E_1$  in Dependence on Temperature  $t$  and Relative Viscosity of Fluidized Bed  $E_B$  in Dependence on Relative Velocity of Air  $W$  from Data of Various Authors

— water, ● this study (corundum),  
○ Kramers<sup>3</sup>, ⊕ Matheson, Herbst, Holt<sup>1</sup>,  
● Peters, Schmidt<sup>6</sup>, ⊖ Siemes, Hellmer<sup>10</sup>.



in a wide range of relative velocities of air or temperatures of water. The literature viscosities which deviate considerably from this course are not plotted in Fig. 2. These are viscosities which are significantly above<sup>4,5</sup> or below<sup>2,8</sup> the curve for the relative viscosity of water.

As the dependence of relative water viscosity on temperature given by Eq. (14) corresponds well with that for relative viscosity of the fluidized bed of corundum it was possible to obtain by use of Eq. (14) the concrete form of relation (13). In literature<sup>24</sup>, for this dependence is given the equation

$$1/\eta_1 = 2.1482\{(t - 8.435) + [8078.4 + (t - 8.435)^2]^{1/2}\} - 120 \quad (21)$$

where  $\eta_1$  is in [P] and  $t$  in [°C].

By multiplication of Eq. (21) by the viscosity of water at temperature  $t = t_0$  the relation is obtained for the relative viscosity of water  $E_1$ . By its combination with Eq. (17) the relation for expressing the dependence of relative viscosity of the fluidized bed  $E_B$  on relative velocity of the fluidizing fluid  $W$  is obtained in the form

$$1/E_B = 2.1482\eta_{10}\{(KW + q - 8.435) + [8078.4 + (KW + q - 8.435)^2]^{1/2}\} - 120\eta_{10} \quad (22)$$

where

$$q = t_0 - KW_0. \quad (23)$$

With respect to our choice of values  $t_0 = 60^\circ\text{C}$  and  $W_0 = 2.5$  it is  $\eta_{10} = 4.688 \cdot 10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$  and  $K = 40^\circ\text{C}$  and thus  $q = -40^\circ\text{C}$ . Eq. (22) after rearrangement becomes

$$1/E_B = 0.403\{(W - 1.21) + [5.049 + (W - 1.21)^2]^{1/2}\} - 0.563, \quad [W_0 = 2.5]. \quad (24)$$

This equation enables calculation of relative viscosity of the fluidized bed  $E_B$ . For only one known viscosity  $\eta_B$  even the absolute viscosities of the fluidized bed can be determined for various velocities of air  $W$ .

The found agreement of dependences  $E_B = E_B(W)$  and  $E_1 = E_1(t)$  prove – in the range of the given accuracy and in the range of experimental conditions – the identity of mathematical form of dependences  $\eta_B = \eta_B(W)$  and  $\eta_1 = \eta_1(t)$ . From this point of view and with the given limitation thus the fluidized bed is analogical with the Newtonian liquid.

If instead of relative velocity of air  $W$  the dimensionless velocity  $U$  defined by Eq. (18) is used, the relative viscosities of the fluidized bed have larger spread of experi-

mental points around the curve for the relative viscosity of water as it is obvious from Fig. 3. In this figure there are not — as in Fig. 2 — plotted values of the relative viscosity calculated from literature<sup>3</sup>. The dimensionless velocities  $U$  for sand as the evaluated material with the mean particle size  $d = 0.127$  mm are considerably smaller than the value  $U_0$  necessary for calculation of the reference viscosity  $\eta_{B0}$  and thus of the relative viscosity  $E_B$ .

It is obvious from Fig. 3 that the relatively best agreement with the course of viscosity of water give the relative viscosities of the fluidized bed obtained in our measurements with nearly spherical particles of corundum.

Greater deviations in courses of both these dependences in comparison to those in Fig. 2 can be the result of inaccuracies in determination of the quantity  $U$ . The elutriation velocity  $w_e$  of material necessary for calculation of the dimensionless velocity  $U$  is not given by any author and was thus calculated. So determined elutriation velocity is perhaps affected by considerable error especially for non-spherical particles.

In a similar way as for the use of the relative velocity  $W$  it is possible to derive the relation between the relative viscosity of the fluidized bed and the dimensionless velocity  $U$ : For  $t_0 = 60^\circ\text{C}$  and  $U_0 = 0.1$  it is  $\eta_{t_0} = 4.688 \cdot 10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$ ,  $K' = 600^\circ\text{C}$ . The following equation is then obtained

$$\frac{1}{E_B} = 6.042\{(U - 0.014) + [0.022 + (U - 0.014)^2]^{1/2}\} - 0.563, \quad [U_0 = 0.1], \quad (25)$$

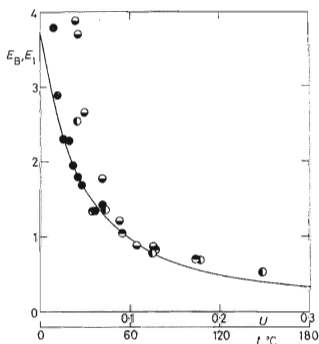


FIG. 3

Relative Viscosity of Water  $E_l$  in Dependence on Temperature  $t$  and Relative Viscosity of Fluidized bed  $E_B$  on Dimensionless Velocity of Air  $U$  from Data of Various Authors

Symbols used are the same as those in Fig. 2.

according to which the relative viscosity of the fluidized bed can be calculated on the basis of the dimensionless velocity  $U$ . But it is necessary to stress that both relations (24) and (25) were obtained for low fluidized beds and that they cannot be used with a great reliability for higher beds. In the range of experimental conditions an analogy between the liquid and the fluidized bed flow can be accepted. This enables description of flow of the fluidized bed *i.e.* of disperse systems as the single-phase flow.

## LIST OF SYMBOLS

$a$	exponent in Eq. (5)
$A$	constant in Eq. (5)
$b$	width of the open channel or airslide
$B$	quantity defined by Eq. (9)
$B_0$	quantity $B$ at the reference velocity of the fluidizing fluid $W = W_0$ or $U = U_0$
$E$	relative viscosity defined by Eqs (11) and (15)
$f$	friction factor
$g$	gravitational acceleration
$h$	height of fluidized bed or liquid at uniform flow
$K$	constant in Eq. (17)
$K'$	constant in Eq. (19)
$\dot{m}$	mass flow rate
$q$	quantity defined by Eq. (23)
$r_h$	hydraulic radius
$R_B$	quantity defined by Eq. (7)
$Re_B$	Reynolds number defined by Eq. (6)
$t$	temperature
$t_z$	parameter of the Student's distribution
$u$	velocity
$\bar{u}_s$	mean velocity of granular material
$U$	quantity defined by Eq. (18)
$w$	superficial velocity of fluidizing fluid
$w_e$	superficial velocity of fluidizing fluid at incipient elutriation of solid particles
$w_p$	superficial velocity of fluidizing fluid at incipient fluidization
$W$	relative velocity of fluidizing fluid
$W_c$	relative velocity of fluidizing fluid at incipient elutriation of solid particles
$\beta$	angle of inclination of the channel bottom or airslide grid
$\eta$	dynamic viscosity
$\rho$	density
$\tau$	shear stress in Eq. (1) or time

## Subscripts

B	related to the fluidized bed
l	related to the liquid phase
0	reference value
s	related to solid phase

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